PAQ-003-001543 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October / November - 2018

Statistics : S - 502

(Mathematical Statistics) (Old Course)

Faculty Code : 003

Subject Code : 001543

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Question - 1 carries 20 marks.

(3) Question - 2 and Question - 3 carry 25 marks.

(4) Students can use their own scientific calculators.

1 Filling the blanks and short questions : (Each 1 mark) **20**

(1) The range of partial regression coefficient is __________.

(2) __________ is a characteristic function of Binomial distribution.

(3) __________ is a characteristic function of Chi-square distribution.

(4) t – distribution with 1 d.f. reduces to __________.

(5) If x follows Gama distribution with parameter p, then $\mu_4 = k_4 + 3k_2^2$ is __________.

(6) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$, then $X_1 + X_2$ is distributed as __________.

(7) A linear combination of independent normal variates is also __________.

(8) For Normal distribution $\mu_4 = k_4 + 3k_2^2$ is __________.

(9) If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2, respectively, then the distribution of $\frac{\chi_1^2 / n_1}{\chi_2^2 / n_2}$ is __________.

(10) If two independent variates $X_1 \sim \gamma(n_1)$ and $X_2 \sim \gamma(n_2)$ then $\frac{X_1}{X_1 + X_2}$ is distributed as ____________.

(11) Measured of Kurtosis coefficient for Normal distribution are ____________.

(12) Weibull distribution has application in ____________.

(13) If $X \sim N(0,1)$ and $Y \sim \chi_n^2$, then statistic $\frac{\sqrt{n}X}{\sqrt{Y}}$ is distributed as ____________.

(14) A measure of linear association of a variable say, X_1 with a number of other variables X_2, X_3, X_4, ... , X_k is known as ____________.

(15) ____________ is a characteristic function of Normal distribution.

(16) Define Bivariate Normal distribution.

(17) Define Beta second kind distribution.

(18) Write mean and variance of Gama distribution with parameter (α, p).

(19) Write mean and variance of Normal distribution.

(20) Write mean and variance of Gama with parameter p distribution.

2 (a) Write the answers any **three** : (each 2 marks) **6**

(1) Define Beta first kind distribution.

(2) Why characteristic function need ?

(3) Define Convergence in Probability.

(4) Prove that, if $r_{12} = r_{13} = r_{23} = \rho$, then $r_{31.2} = \frac{\rho}{1+\rho}$

(5) Prove that $b_{12.3} = \frac{b_{12} - b_{13}\, b_{23}}{1 - b_{13}\, b_{23}}$

(6) In trivariate distribution it is found that $r_{12} = 0.77$, $r_{13} = 0.72$ and $r_{23} = 0.52$. Find (i) $r_{12.3}$, and (ii) $R_{1.23}$.

(b) Write the answers any **three** : (each 3 marks) **9**

(1) Usual notation of multiple correlation and multiple regression, prove that $\sigma_{1.23}^2 = \sigma_1^2\left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$.

(2) Obtain Probability density function for the characteristic function $\phi_X(t) = e^{-\left(\frac{1}{2}t^2\sigma^2\right)}$.

(3) Define Exponential distribution and obtain its MGF. From MGF obtain its mean and variance.

(4) Define truncated Binomial distribution and also obtain its mean and variance.

(5) Usual notation of multiple correlation and multiple regression, prove that $b_{12} = \frac{b_{12.3} + b_{13.2}\, b_{32.1}}{1 - b_{13.2}\, b_{31.2}}$.

(6) Prove that $\mu_r = (-i)^r \left[\frac{d^r}{dt^r} \phi_u(t) \right]_{t=0}$; where $u = x - \mu$.

(c) Write the answers any **two** : (each 5 marks) **10**

(1) Obtain marginal distribution of x for Bi-variate distribution.

(2) State and Prove that Chebchev's inequality.

(3) If x and y are independent χ^2 variates with n_1 and n_2 degree of freedom respectively then obtain distribution of $\frac{x}{x+y}$ and $x+y$.

(4) Drive t-distribution.

(5) Usual notation of multiple correlation and multiple regression, prove that $R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}$.

3 (a) Write the answers any **three** : (each 2 marks) **6**

(1) Define Log Normal distribution when $Y = \log_e (x-a)$.

(2) Obtain characteristic function of Poisson distribution with parameter λ.

(3) Obtain relation between t and F distribution.

(4) Prove that $\phi_X(0) = 1$ and $|\phi_X(t)| \le 1$.

(5) Prove that,

$$\sigma_{3.12}^2 = \frac{\sigma_3^2 \left(1 - r_{12}^2 - r_{23}^2 - r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}\right)}{\left(1 - r_{12}^2\right)}$$

(6) In trivariate distribution it is found that $\sigma_1 = 2$, $\sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$.
Find (i) $b_{13.2}$ and (ii) $\sigma_{3.12}$.

(b) Write the answers any **three** : (each 3 marks) **9**

(1) Define truncated Poisson distribution and also obtain its mean and variance.

(2) Obtain Harmonic mean of $X \sim \gamma(\alpha, p)$.

(3) Obtain mean and variance of Uniform Distribution.

(4) Prove that $\mu_r' = (-i)^r \left[\frac{d^r}{dt^r} \phi_X(t)\right]_{t=0}$

(5) Usual notation of multiple correlation and multiple regression, prove that $r_{xy} + r_{yz} + r_{xz} \geq -\frac{3}{2}$.

(6) Usual notation of multiple correlation and multiple regression, prove that $b_{12.3} b_{23.1} b_{31.2} = r_{12.3} r_{23.1} r_{31.2}$.

(c) Write the answer any **two** : (each 5 marks) **10**

(1) Obtain relation between F and χ^2.

(2) Drive Normal distribution.

(3) Obtain MGF of Gamma distribution with parameters α and p. Also show that $3\beta_1 - 2\beta_2 + 6 = 0$.

(4) If the joint pdf of x and y is

$$f(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2\left(1-\rho^2\right)}\left\{x^2 - 2\rho xy + y^2\right\}}$$

where $-\infty \leq x, y \leq \infty; -1 \leq \rho \leq 1$,

then find

(i) Marginal distribution of y.

(ii) Conditional distribution of x when y is given.

(5) Usual notation of multiple correlation and multiple regression, prove that $r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$